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NEW INSIGHTS INTO THE PRODUCTION OF HEAVY QUARKONIUM*

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ABSTRACT

Recent data from the Tevatron have revealed that the production rate of prompt charmonium is more than an order of magnitude larger than the best theoretical predictions of a few years ago. This surprising result can be understood by taking into account new production mechanisms that include fragmentation and the formation of charmonium from color-octet $c\bar{c}$ pairs.

1. Introduction

The typical high energy physics conference these days includes talk after talk showing remarkable agreement between experiment and theory. There is an occasional two-sigma discrepancy, but most such problems will go away if you have the patience to wait for better data. However there is one problem where experimental results have differed from theoretical predictions by orders of magnitude. This problem is the production of charmonium at large transverse momentum at the Tevatron.

2. Color-singlet Model

Until recently, the conventional wisdom on the production of heavy quarkonium was based primarily on the *color-singlet model*.[1] In this model, the cross section for producing a charmonium state is proportional to the perturbative cross section for producing a color-singlet $c\bar{c}$ pair with vanishing relative momentum and with appropriate angular-momentum quantum numbers: $^1S_0$ for η_c, $^3S_1$ for J/ψ and ψ', ^3P_J for χ_{cJ}, etc. The color-singlet model has great predictive power. The cross section for producing a quarkonium state in any high energy process is predicted in terms of a single nonperturbative parameter for each orbital-angular-momentum multiplet. The nonperturbative factor is $|R(0)|^2$ for S-wave states, $|R'(0)|^2$ for P-wave states, etc., where $R(r)$ is the radial wavefunction. For example, the inclusive differential

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cross sections for producing J/ψ and χ_{cJ} in the color-singlet model have the form

$$d\sigma(\psi + X) = d\widehat{\sigma}(c\bar{c}(\underline{1}, {}^3S_1) + X)\,|R_\psi(0)|^2, \tag{1}$$

$$d\sigma(\chi_{cJ} + X) = d\widehat{\sigma}(c\bar{c}(\underline{1}, {}^3P_J) + X)\,|R'_{\chi_c}(0)|^2. \tag{2}$$

As the name suggests, the color-singlet model is not a complete theory of quarkonium production derived from QCD. The model ignores relativistic corrections, which take into account the nonzero relative velocity v of the quark and antiquark. These corrections may be numerically significant, since the average value of v^2 is only about 1/3 for charmonium and 1/10 for bottomonium. The color-singlet model also assumes that a $c\bar{c}$ pair produced in a color-octet state will never bind to form charmonium. This assumption must break down at some level, since a color-octet $c\bar{c}$ pair can make a nonperturbative transition to a color-singlet state by radiating a soft gluon. The clearest evidence that the color-singlet model is incomplete comes from radiative corrections. In the case of S-waves, these can be calculated consistently within the color-singlet model. However, in the case of P-waves, the radiative corrections contain infrared divergences that cannot be factored into $|R'(0)|^2$. This problem was first noted in 1976 in connection with the decays of χ_c states,[2] but it was solved only recently.[3] The divergence arises from the radiation of a soft gluon from either the quark or the antiquark that form the color-singlet ${}^3P_J$ bound state. The infrared divergence can be factored into a matrix element $\langle \mathcal{O}_8^{\chi_c}({}^3S_1)\rangle$ that is proportional to the probability for a pointlike $c\bar{c}$ pair in a color-octet ${}^3S_1$ state to form χ_c plus anything. Thus, perturbative consistency demands that the formula (2) of the color-singlet model be modified to take into account the nonzero probability for a $c\bar{c}$ pair produced in a color-octet state to bind to form charmonium:

$$\begin{aligned} d\sigma(\chi_{cJ} + X) &= d\widehat{\sigma}(c\bar{c}(\underline{1}, {}^3P_J) + X)\,|R'_{\chi_c}(0)|^2 \\ &\quad + (2J+1)\, d\widehat{\sigma}(c\bar{c}(\underline{8}, {}^3S_1) + X)\,\langle \mathcal{O}_8^{\chi_c}({}^3S_1)\rangle. \end{aligned} \tag{3}$$

The color-singlet model can be used to predict the production rate of charmonium at large transverse momentum in hadron colliders. The first thorough treatment of this problem was given by Baier and Rückl in 1981, and their analysis remained the conventional wisdom for the next decade.[4] A ψ with large p_T can be produced either directly, or from a χ_{cJ} with large p_T that decays via $\chi_{cJ} \to \psi\gamma$, or by the decay of a B hadron with large p_T. Baier and Rückl assumed that the direct production of charmonium is dominated by the parton processes that are lowest order in the QCD coupling constant α_s. The relevant parton processes that produce $c\bar{c}$ pairs at large p_T are $gg \to c\bar{c} + g$, $gq \to c\bar{c} + q$, $g\bar{q} \to c\bar{c} + \bar{q}$, and $q\bar{q} \to c\bar{c} + g$. The cross sections $d\widehat{\sigma}$ for these processes are all of order α_s^3, but they have different dependences on p_T. The only parton process that produces direct ψ is $gg \to c\bar{c} + g$, and it gives a cross section that has the behavior $d\widehat{\sigma}/dp_T^2 \sim 1/p_T^8$ at large p_T. The dominant parton process for direct χ_{cJ} is $gg \to c\bar{c} + g$, and it gives $d\widehat{\sigma}/dp_T^2 \sim 1/p_T^6$. Both of these

cross sections fall more rapidly with p_T than typical jet production cross sections, which behave like $d\hat{\sigma}/dp_T^2 \sim 1/p_T^4$. The cross section for b quark production has this scaling behavior when $p_T \gg m_b$. Thus the conventional wisdom was that ψ's at large p_T should come predominantly from b quarks, with direct χ_{cJ}'s being the next most important source, and direct ψ's being negligible. This conventional wisdom has been completely overthrown by recent experimental data from the Tevatron.

3. Prompt Charmonium at the Tevatron

In the 1988-89 run of the Tevatron, the cross section for ψ production at large p_T was measured by the CDF collaboration. They also measured the cross section for χ_c at large p_T. Assuming the conventional wisdom that ψ's at large p_T come predominantly from b quarks and from direct χ_c's, they inferred that the fraction f_b of ψ's that come from b quarks was about 60%.[5] This number was then used to determine the b-quark cross section. Unfortunately, the conventional wisdom proved to be wrong. The fraction f_b is actually closer to 15%, and the b quark cross section is significantly smaller than the result obtained in the 1988-89 run.

The breakthrough came in the 1992-93 run of the Tevatron, with the installation of a silicon vertex detector at CDF. This device can be used to measure the separation between the collision point of the p and $\bar{p}$ and the point where the ψ decays into leptons. If a ψ with large p_T is produced by QCD mechanisms, then the leptons from its decay will trace back to the $p\bar{p}$ collision point and the ψ is called *prompt*. Similarly, if a χ_c is produced by QCD mechanisms and decays radiatively, the resulting ψ is also prompt. On the other hand, the ψ's coming from b quarks are not prompt. A B hadron with large p_T will travel a distance on the order of a millimeter before it decays weakly. Thus the leptons from the decay of the ψ will trace back to a vertex with a measureable displacement from the collision point.

By using the vertex detector to remove the background from b quarks, CDF was able to study the QCD production mechanisms directly. They discovered that the cross section for prompt ψ production at large p_T is about 30 times larger than predicted.[5] The prompt cross section for $\chi_{c1} + \chi_{c2}$ is also larger than predicted by a similar factor. The cross section for prompt ψ' at large p_T is three orders of magnitude larger than the theoretical prediction. The conventional wisdom on prompt charmonium production fails completely when confronted with this data. The data can only be explained by new production mechanisms.

4. Fragmentation

Braaten and Yuan pointed out in 1993 that the dominant production mechanism for charmonium at sufficiently large p_T must be *fragmentation*, the formation of charmonium within the jet initiated by a parton with large transverse momentum.[6] This

production mechanism had not been included in any previous theoretical predictions for $p\bar{p}$ collisions. The importance of fragmentation should have been realized long ago. There are factorization theorems of perturbative QCD dating back to 1980 that guarantee that the inclusive production of a hadron at large p_T is dominated by fragmentation.[7] According to these theorems, the asymptotic form of the inclusive differential cross section for producing a hadron H with momentum P is

$$d\sigma(H(P)+X) \;=\; \sum_i \int_0^1 dz\; d\widehat{\sigma}(i(P/z)+X)\; D_{i\to H}(z)\;, \tag{4}$$

where $d\widehat{\sigma}$ is the differential cross section for producing a parton of type i with momentum P/z and $D_{i\to H}(z)$ is a fragmentation function. It gives the probability that a jet initiated by parton i includes a hadron H carrying a fraction z of the parton momentum. The parton cross sections $d\widehat{\sigma}$ can be calculated using perturbation theory (up to parton distributions for the colliding particles if they are hadrons). All the nonperturbative dynamics involved in the formation of the hadron H is contained in the fragmentation functions.

The factorization theorem (4) applies equally well to heavy quarkonium as to light hadrons. It is easy to see that the conventional calculations of ψ production at large p_T did not include fragmentation contributions. At large p_T, the parton cross sections $d\widehat{\sigma}$ in (4) have the scaling behavior $d\widehat{\sigma}/dp_T^2 \sim 1/p_T^4$ up to logarithmic corrections. However the leading-order cross sections for $gg \to \psi + g$ and $gg \to \chi_{cJ} + g$ in the color-singlet model behave asymptotically like $1/p_T^8$ and $1/p_T^6$, respectively. So where are the fragmentation contributions? The answer is that they do appear in the color-singlet model, but only at higher order in perturbation theory. They appear at next-to-leading order for χ_c and at next-to-next-to-leading order for ψ. These higher-order corrections would be prohibitively difficult to calculate in their entirety, but fortunately the fragmentation functions can be calculated relatively easily. The fragmentation functions that are needed for prompt ψ production in $p\bar{p}$ collisions, including $g \to \psi$, $c \to \psi$, $g \to \chi_{cJ}$, $c \to \chi_{cJ}$, and $\gamma \to \psi$, have all been calculated to leading order in α_s.[8] The fragmentation functions for ψ were calculated in the color-singlet model. The fragmentation functions for χ_{cJ} were calculated using the factorization formula (3), which includes a color-octet contribution.

To calculate the fragmentation contribution to the cross section for prompt ψ production at large p_T in $p\bar{p}$ collisions, the fragmentation functions $D_{g\to H}(z)$ and $D_{c\to H}(z)$ must be folded with parton scattering cross sections for $ij \to g + k$ and for $ij \to c + k, \bar{c} + k$, respectively. They must then be convoluted with parton distributions $f_{i/p}(x_1)$ and $f_{j/\bar{p}}(x_2)$ for the colliding p and $\bar{p}$. The largest contribution comes from the color-octet term in the fragmentation function for $g \to \chi_c$. The fragmentation contribution increases the theoretical prediction by an order of magnitude, bringing it to within a factor of 3 of the CDF data.[9] Given the many sources of uncertainty in

the calculation, this can be considered reasonable agreement.

5. Color-octet Mechanism

While fragmentation may explain the CDF data on prompt ψ production, it is not enough to explain the data on prompt ψ''s. In this case, the conventional wisdom gave predictions that were 3 orders of magnitude smaller than the data. When fragmentation is included, the prediction increases by more than an order of magnitude at large p_T, but it remains more than an order of magnitude below the data. The big difference between ψ and ψ' is that the ψ signal is fed by the decay of χ_c's, but there are no known charmonium states that feed the ψ' signal.

One possible explanation for this ψ' anomaly at CDF is that the ψ' signal is fed by the decays of higher charmonium states that have not yet been discovered.[10] Among the candidates are D-wave states, higher P-wave states, and $c\bar{c}g$ states. The main difficulty with this solution is to explain how these states can have such a dramatic effect on ψ' production at the Tevatron and not have shown up in any charmonium experiments at lower energy.

Another possibility is that the solution to the ψ' anomaly lies in a new production mechanism. Braaten and Fleming have proposed that the ψ''s come primarily from a color-octet term in the gluon fragmentation function for ψ'.[11] The basis for this proposal is a general theory of inclusive quarkonium production that was recently developed by Bodwin, Braaten, and Lepage.[12] This approach allows one to calculate not only perturbative corrections to any order in α_s, but also relativistic corrections to any order in v^2. The inclusive cross section for producing a quarkonium state H satisfies a factorization formula of the form

$$d\sigma(H+X) \;=\; \sum_n d\widehat{\sigma}(c\bar{c}(n)+X)\;\langle\mathcal{O}_n^H\rangle, \tag{5}$$

where $d\widehat{\sigma}$ is the inclusive cross section for producing a $c\bar{c}$ pair separated by a distance less than $1/m_c$ and in a color and angular-momentum state labelled by n. Since $d\widehat{\sigma}$ depends only on short distances, it can be calculated using perturbative QCD. The nonperturbative factor $\langle\mathcal{O}_n^H\rangle$ in (5) is proportional to the probability for a pointlike $c\bar{c}$ pair in the state n to form the bound state H. It can be defined rigorously as a matrix element in nonrelativistic QCD. The relative importance of the various terms in (5) can be estimated using scaling rules that tell how the matrix elements scale with m_c and v.

The factorization formula (5) also applies to fragmentation functions, which have the general form

$$D_{i\to H}(z) \;=\; \sum_n d_{i\to n}(z)\;\langle\mathcal{O}_n^H\rangle. \tag{6}$$

For example, the process $g\to c\bar{c}$ produces a $c\bar{c}$ pair in a color-octet ${}^3S_1$ state and

gives rise to a term in the gluon fragmentation function of order α_s:

$$D_{g\to H}(z) \;=\; \frac{\pi\alpha_s}{24m_c^3}\delta(1-z)\,\langle\mathcal{O}_8^H(^3S_1)\rangle. \tag{7}$$

This term was included in the fragmentation calculations for direct χ_c production, because it is required for perturbative consistency. It was not included in the calculations for the S-wave states ψ and ψ' on the grounds that the matrix element $\langle\mathcal{O}_8(^3S_1)\rangle$ is suppressed by v^4 relative to the nonperturbative factor $|R(0)|^2$ in the leading color-singlet term. However the term (7) may well be numerically important, because the leading color-singlet term is suppressed by a short-distance factor $d_n(z)$ that is of order α_s^3. The CDF data on ψ' production can be explained by including the term (7) in the gluon fragmentation function for ψ' and adjusting the matrix element $\langle\mathcal{O}_8^{\psi'}(^3S_1)\rangle$ to fit the data.[11] The term (7) must also appear in the gluon fragmentation function for ψ, and it may explain the CDF data on direct ψ's that do not come from χ_c's.[13,14]

6. Recent Developments

Recently, there have been several significant experimental developments in charmonium at large p_T at the Tevatron. The CDF collaboration has measured the ratio of the prompt cross sections for χ_{c1} and χ_{c2},[16] providing one more test of our understanding of charmonium production mechanisms. They have also measured the cross sections for the bottomonium states Υ, Υ', and Υ''.[16] This information is valuable, because the larger mass of the bottom quark changes the relative importance of the various production mechanisms.

There have also been important developments in theory. The next-to-leading-order perturbative corrections to the fragmentation function (7) have been calculated by Ma.[15] Eventually all the relevant fragmentation functions should be calculated to next-to-leading order in α_s. Another important development is that Cho and Leibovich have calculated the terms in the parton cross sections for $ij \to c\bar{c} + k$ that correspond to the matrix element $\langle\mathcal{O}_8^H(^3S_1)\rangle$.[14] They used these results to extend the calculations of prompt charmonium production down to moderate p_T and also to calculate the Υ production rate at the largest values of p_T measured at the Tevatron. Ultimately, one would like to extend the theoretical predictions all the way down to $p_T = 0$. It may be necessary to include other color-octet matrix elements in the factorization formula (5) for the cross section at low p_T. It will also be necessary to resum the effects of soft gluons in order to get a cross section that vanishes properly at $p_T = 0$.

In conclusion, data from the Tevatron is driving us toward a deeper understanding of the production of heavy quarkonium. The order-of-magnitude discrepancies between experiment and theory can be understood by introducing new production mechanisms that are based on a theory of inclusive quarkonium production derived

from QCD. I believe that we are on our way to a comprehensive description of quarkonium production in all high energy processes in terms of the quark mass, the QCD coupling constant, and a few well-defined phenomenological matrix elements.

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